Real Analysis Part III
PAPER V – NOTES-6

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Functions of two variables

If x, y are two independent variables ~~and~~ a variable Z depends for its values on the values of x, y by a functional Relation

$$Z = f(x,y) \quad — \quad \text{I}$$

then we say Z is a function of x, y.

The function represented by Equation (1) is an explicit function of two variables.

The Neighbourhood of a point.

The Set of values x_1, y_1 other than a, b that satisfy the condition

$$|x_1 - a| < \delta, \ |y_1 - b| < \delta.$$

where δ is an arbitary Small positive number, is said to form a neighbourhood of the point (a, b)

Thus a neighbourhood is the Square

$[a-\delta, \ a+\delta \ ; \ b-\delta, \ b+\delta]$

where x takes any value from $a-\delta$ to $a+\delta$ except a and y from $b-\delta$ to $b+\delta$ except b.

<u>Limit Point</u> A point (ξ, η) is called a limit point or a point of condensation of a set of points S, if every nbhd of (ξ, η) contains an infinite number of points of S. The limit point itself may or may not be a point of the set.

For example the point $(0,0)$ is a limit point of the set $\{(1/m, 1/n) ; m, n \in N$.

<u>The limit of a function</u>. A function f is said to tend to a limit l as a point (x, y) tends to the point (a, b) if to every arbitrary small positive number ϵ, there correspond a positive number δ: such that

$$|f(x,y) - l| < \epsilon$$

for every point (x,y) [different from (a,b)] which satisfies

$$|x-a| < \delta, \quad |y-b| < \delta .$$

IN OTHER WORDS A function f tends to a limit l when (x,y) tends to (a,b) if to every positive number ϵ, there corresponds a neighbourhood N of (a,b) such that $|f(x,y) - l| < \epsilon$

for every point (x,y). ~~[different from (a,b)]~~ which satisfies

$$\cancel{|x-a| < \delta, \; |y-b| < \delta}$$

Other than (a,b) of the nbd N

Symbolically we then write.

$$\lim_{(x,y)\to(a,b)} f(x,y) = l$$

l is the limit (the double limit or the simultaneous limit) of f when x,y tend to a,b simultaneously.